Prediction of Flow Resistance in an Open Channel over Movable Beds Using Artificial Neural Network

Satish Kumar[1]; Jnana Ranjan Khuntia, Ph.D.[2]; and Kishanjit Kumar Khatua, Ph.D.[3]

Abstract: Estimating flow resistance is essential for the hydraulic analysis of a river and the evaluation of conveyance in a specific flow condition. Under bed-load transport conditions, the resistance to the flow in an open channel is different from fixed-bed condition and requires a distinct method for its evaluation. The geometric and hydraulic parameters influence flow resistance characteristics in the mobile bed load. In the present study, a wide range of experimental flume data sets are investigated to derive the dependency of the dimensionless parameters on the flow resistance under mobile bed-load conditions. The five most important dimensionless parameters, such as relative submergence depth, bed slope, aspect ratio, Reynolds number, and Froude number, are suggested because they show a unique relationship to the dependent parameter. An artificial neural network (ANN) model to predict the flow resistance is proposed by considering these independent parameters as the input parameters. To verify the strength of the model, the performances of previous researchers' models were also evaluated and compared with the present work by considering a wide range of data sets. It is found that the previous models can be used for a specific range of data sets only, whereas the proposed ANN-based model is capable of performing well for a wide range of geometric and hydraulic conditions of a channel. **DOI:** 10.1061/(ASCE)HE.1943-5584.0002085. *© 2021 American Society of Civil Engineers.*

Author keywords: Flow resistance; Artificial neural network; Open channels; Independent parameter.

Introduction

Investigating the effect of moving bed load on flow parameters of an open-channel flow has gained importance in the last few decades. The bed load consists of various sediment particles resting at the bed and are intermittently entrained in the turbulent water flow. While carried along the direction of flow, these are deposited a short distance downstream of the channel. The transport of bed load depends on the flow strength and size and density of the sediment particles. Smaller and lighter sediment particles are more likely to be carried for longer distances, while larger or denser particles will fall through the flow. Thus, the movement of bed load is caused by rolling, sliding, and saltation of grains along the bed. Sediments, in turn, offer resistance to flow, causing its retardation. In the case of open-channel flow with a rigid boundary, the resistance is specified by a constant roughness coefficient, and a resistance formula can be applied directly for the computation of flow parameters. But in the case of a mobile bed, the flow resistance is influenced both by grain or skin friction, and the traditional resistance formula cannot be applied directly without knowing how the resistance coefficient will change under different flow and sediment conditions.

Many researchers have explained the effect of bed-load transport in terms of the extra resistance caused by bedforms. The direct impact of the bed load on flow resistance for different bed slope conditions was reported by Bathurst et al. (1982). Wiberg and Rubin (1989) observed that the flow resistance associated with bed-load transport conditions could reach much higher values than those observed with fixed-bed conditions. Song et al. (1998) observed that the bed-load movement that increases the flow resistance depends on the volumetric sediment concentration of the bed load and the size of the moving particles. Carbonneau and Bergeron (2000) observed the wakes that shed as particles are accelerated by the strength of flow and produce a layer of roughness that develops well past the highest point of the saltation layer, affecting the mean velocity profile. Hu and Abrahams (2004) observed that the friction factor due to the bed-load movement could be 22.06% of total resistance and reported that flow resistance is affected by factors such as sediment concentration, relative submergence depth, particle diameter, bed slope, and Froude number. Gao and Abrahams (2004) considered their experimental data as well as that of Song et al. (1998). They observed that the sediment concentration, relative roughness, and dimensionless particle size are the factors that influence the flow resistance. Hu and Abrahams (2005) investigated that bed-load grains colliding with mobile beds lose more momentum to the bed than grains colliding with fixed beds. They concluded that the resistance due to flow in a moving bed load becomes higher than that in fixed beds. Recking et al. (2008) reported that friction factor f increases with the sediment concentration in mobile bed conditions. Kumar (2011) investigated the flow resistance in alluvial channels and developed an equation for friction factor by analyzing its dependency on Reynolds and Froude numbers along with particle relative roughness. Omid et al. (2010) investigated the effect of bed-load transport on flow resistance of alluvial channels experimentally by varying the particle size. They observed that transport of fine particles could decrease the friction factor by 22% and 24% for smooth and rough beds, respectively. Ferro (2018) calibrated the model parameters of flow velocity, water depth, and bed slope of the

[1]Ph.D. Scholar, Dept.of Civil Engineering, National Institute of Technology Rourkela, Rourkela 769008, India (corresponding author). Email: satishserly@gmail.com

[2]Assistant Professor, Dept. of Civil Engineering, St. Martin's Engineering College, Secunderabad, Telangana 500100, India; formerly, Ph.D. Scholar, Dept. of Civil Engineering, National Institute of Technology Rourkela, Rourkela 769008, India. ORCID: https://orcid.org/0000-0003-3943-4220. Email: jnanaranjan444@gmail.com

[3]Professor, Dept. of Civil Engineering, National Institute of Technology Rourkela, Rourkela 769008, India. Email: kkkhatua@nitrkl.ac.in

Note. This manuscript was submitted on May 12, 2020; approved on January 4, 2021; published online on March 13, 2021. Discussion period open until August 13, 2021; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Hydrologic Engineering***, © ASCE, ISSN 1084-0699.

experimental data of Recking (2006) and performed the dimensional analysis to obtain an equation for flow resistance. Hou et al. (2019) conducted experiments in steep gradient channels with various particle sizes. They found that the flow resistance decreases with an increase in Froude number (Fr) and inversely with longitudinal bed slope. From the different investigations, it is seen that the flow resistance of a bed-load channel is increased, decreased, or unchanged due to different sizes of bed-load materials, bed slope, and turbulent flow properties over a mobile bed that differ significantly from those over a fixed bed (Campbell et al. 2005). Therefore, it can be stated that the resistance to flow in the mobile bed-load condition depends strongly on both the flow conditions and bed-load particles (Garcia 2008). The traditional resistance formula cannot be applied directly without knowing how the resistance coefficient will change under different flow and sediment conditions (Yang and Lim 2003). Besides, bed load forms a layer that alters the roughness height of bed particles, while suspended load may affect turbulence, and hence flow resistance. Many researchers have analyzed flow conditions over the mobile bed for gravel bed streams and proposed equations for Darcy-Weisbach friction factor f for limited geometrical and hydraulic conditions. Therefore, reliable modeling of the roughness coefficient for a wide range of flow conditions is necessary for evaluating the conveyance and the hydraulic analysis of streams.

The objective of the present work is to evaluate the flow resistance of a gravel bed channel by analyzing its dependencies on the five most influential dimensionless parameters derived from the dimensional analysis of important variables pertaining to flow in movable bed conditions. Using these dimensionless terms, an artificial neural network (ANN) model is also proposed to predict the flow resistance. The five influential parameters considered as input have shown reasonable nonlinear relationships with the friction factor individually. Model performances of previous researchers were also tested and compared with the strength of the present model considering a wide range of data sets. Because the previous models are valid for a specific range of data sets, it is confirmed from the statistical analysis that the proposed ANN-based model is capable of performing well for a wide range of geometric and hydraulic conditions of a gravel bed channel. The study provides an improved flow resistance prediction.

Methodology

The flow resistance equation for computing friction factor f applicable to the condition of uniform flow in an open channel is represented in the following form:

$$\sqrt{\frac{8}{f}} = \frac{U}{\sqrt{gRS_o}} = \frac{U}{u_*} \tag{1}$$

where f = friction factor; u_* = shear velocity defined as $u_* = \sqrt{gRS_o} = \sqrt{\tau_o/\rho}$, where τ_o is the boundary shear stress; ρ = density of water; U = mean velocity; R = hydraulic radius; g = acceleration due to gravity; and S_o = longitudinal slope of the bed.

Without bed-load transport conditions where turbulence is fully developed, Keulegan (1938) proposed a logarithmic model and expressed the friction factor in terms of relative submergence depth (R/D) as

$$\sqrt{\frac{8}{f}} = 6.25 + 5.75 \log\left(\frac{R}{D}\right) \tag{2}$$

Manning (1891) and Meyer-Peter and Müller (1948) studied the importance of grain resistance for moving bed load and defined the expression for velocity as

$$u = K_s R^{2/3} S_0^{1/2} \tag{3}$$

where K_s = grain characteristics for bed roughness, defined as

$$K_s = \frac{21.1}{D^{1/6}} \tag{4}$$

The formulation for friction law was given as

$$\sqrt{\frac{8}{f}} = \frac{U}{u_*} = 6.75\left(\frac{R}{D}\right)^{1/6} \tag{5}$$

Cao (1985) conducted experiments in a 0.6-m-wide flume for slopes from 0.5% to 9% in moving bed-load conditions with 44-, 22-, and 11-mm grain diameter. The friction law formulation was proposed as

$$\sqrt{\frac{8}{f}} = 3.75 + 5.91 \log\left(\frac{R}{D}\right) \tag{6}$$

Recking (2006) conducted experiments in a flume by varying longitudinal slope from 1% to 9% and friction factor f was found under equilibrium condition of bed-load transport over uniformly sized gravel particles having a mean particle diameter of 2.3, 4.9, 9, and 12.5 mm. He identified three regimes: no bed-load transport, low bed-load transport, and high bed-load transport. Characterization of each regime was done by distinct friction law as observed in the difference in the movement pattern of sediment particles. He observed that in Regime 1 when no sediment transport, $\sqrt{8/f}$ increases with (R/D); in Regime 2, bed load appears and $\sqrt{8/f}$ is constant; and in Regime 3 both bed load and $\sqrt{8/f}$ increase with (R/D). In the study, different flow resistance equations were reviewed for a wide range of flow conditions, and a logarithmic friction model was proposed for two different ranges of relative submergence depth (R/D) as

$$\sqrt{\frac{8}{f}} = -1 + 9.5 \log\left(\frac{R}{D}\right) \quad \text{for } \frac{R}{D} < 16.9 \tag{7}$$

$$\sqrt{\frac{8}{f}} = 3.6 + 5.75 \log\left(\frac{R}{D}\right) \quad \text{for } \frac{R}{D} > 16.9 \tag{8}$$

Julien (2002) proposed the logarithmic friction model for finding the friction factor as

$$\sqrt{\frac{8}{f}} = 5.75 \log\left(\frac{2R}{D}\right) \tag{9}$$

Recking et al. (2008) fitted logarithmic functions to data sets of Recking (2006) and obtained two distinct formulations for flow condition without and with bed-load transport as

$$\sqrt{\frac{8}{f}} = 3.6 + 3.2\,\ell n\left(\frac{R}{D}\right) \quad \text{for flows without bed load} \tag{10}$$

$$\sqrt{\frac{8}{f}} = 0.67 + 3.2\,\ell n\left(\frac{R}{D}\right) \quad \text{for flows with bed load} \tag{11}$$

J. Hydrol. Eng., 2021, 26(5): 04021015

The friction factor formulation by analytical means is difficult because the interactions between sediment transport, bedform, and flow properties are very complicated processes. Most of the previous flow resistance equations use only a single dimensionless parameter, i.e., relative submergence depth. However, the previous investigations show that the flow resistance is influenced by several other factors such as the relative submergence depth, bed slope, aspect ratio, Reynolds number, and Froude number. Considering the preceding points of view, in the present study, a wide range of experimental data sets were investigated to observe these dimensionless parameters' effects on the flow resistance in case of flow in a mobile bed channel. The flow investigations suggest that flow variables such as velocity, hydraulic radius, viscosity, longitudinal slope, flow depth, channel width, sediment particle size, density of water, and gravitational force are important for determining the flow resistance in movable bed-load condition and thus the friction factor f can be represented as

$$f = \text{function of}\,(U, R, \mu, S_o, D, W, H, D, \rho, g) \tag{12}$$

where U = mean velocity of flow; S_o = longitudinal bed slope; R = hydraulic radius; μ = dynamic viscosity; ρ = density of water; D = mean diameter of the particle; W = width; H = depth of flow; and g = acceleration due to gravity.

From dimensional analysis, these flow variables can be reduced to a set of five important dimensionless terms: (1) R/D = relative submergence height; (2) $U/\sqrt{gR} = Fr$ (Froude number); (3) $(\rho UR)/\mu$ = Re (Reynolds number); (4) S_o = bed slope; and (5) $W/H = \delta$ (aspect ratio). Thus, the previous functional relation can be expressed as

$$f = \text{function of}\left(\frac{R}{D}, Fr, \text{Re}, \delta, S_o\right) \tag{13}$$

Flow in an open channel is affected by force due to gravity and is generally turbulent, hence the effect of Froude number Fr and Reynolds number Re is taken into consideration.

It has been observed that the existing empirical equations of flow resistance of a mobile bed channel generally use one dimensionless parameter, i.e., relative submergence height (R/D), to estimate flow resistance in the form of friction factor. In the present study, other important, influential flow parameters were also considered to develop a new flow resistance model. The model is developed using the wide ranges of experimental data sets of various researchers. The results of the model were compared with that of previous researchers. To achieve this, different types of statistical indicators, such as the mean absolute percentage error (MAPE), mean absolute error (MAE), mean percentage error (MPE), root-mean-square error (RMSE), percent bias (PBIAS), Kling-Gupta efficiency (KGE), and index of agreement (I_d), were computed as given in Eqs. (14)–(20). This statistical analysis facilitates the comparison of previous models with the proposed ANN model and evaluates the acceptability of each of the models concerning the ranges of data sets

$$\text{MAPE} = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{O_i - P_i}{P_i}\right| \tag{14}$$

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|O_i - P_i| \tag{15}$$

$$\text{MPE} = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{O_i - P_i}{A_i}\right) \tag{16}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(O_i - P_i)^2}{N}} \tag{17}$$

$$\text{PBIAS} = 100\,\frac{\sum_{i=1}^{N}(P_i - O_i)}{\sum_{i=1}^{N} O_i} \tag{18}$$

$$\text{KGE} = 1 - \sqrt{(r-1)^2 + (\alpha - 1)^2 + (\beta - 1)^2} \tag{19a}$$

where r = linear correlation between observed and predicted values; α = measure of the flow variability error; and β = bias term, as defined as follows:

$$r = \frac{\sum_{i=1}^{N} P_i O_i - n\overline{P_i}\,\overline{O_i}}{\sqrt{(\sum_{i=1}^{N} P_i^2 - n\overline{P_i}^2)}\sqrt{(\sum_{i=1}^{N} O_i^2 - n\overline{O_i}^2)}};$$

$$\alpha = \frac{\sqrt{(\sum_{i=1}^{N} P_i - \overline{P_i})^2}}{\sqrt{(\sum_{i=1}^{N} O_i - \overline{O_i})^2}}; \qquad \beta = \frac{\overline{P_i}}{\overline{O_i}} \tag{19b}$$

$$I_d = 1 - \frac{\sum_{i=1}^{N}(O_i - P_i)^2}{\sum_{i=1}^{N}(|O_i - \overline{O_i}| + |P_i - \overline{O_i}|)^2} \tag{20}$$

where O_i, $\overline{O_i}$, P_i, $\overline{P_i}$, and N = observed value, mean of the observed value, predicted value, mean of the predicted value, and number of samples, respectively.

Source of the Data Set

The present study includes 1,269 laboratory flume data sets, with various hydraulic and geometric parameters measured in equilibrium mobile bed-load condition as detailed in Recking (2006). The data sets undertaken in this study were selected using the screening criteria of bed slope equal to more than 0.1%, no suspension, and absence of bedforms. The different flow conditions for these data sets are tabulated in Appendix I. Gilbert (1914) did an experimental study and provided 311 data sets in a straight channel by varying the grain size diameter on different channel dimensions and changing the bed slope. Cassey (1935) carried out 78 experimental observations on single-channel geometry by varying the bed slope and grain size. A total of 261 combinations of experimental observations were undertaken by Mavis (1937) for five different grain size diameters on the same channel width for different variations of bed slope. Bogardi and Yen (1939) performed 44 experimental observations for a mobile bed for various channel widths, slopes, and bed materials. Yang (1939) and Meyer-Peter and Müller (1948) carried out 71 and 105 observations of mobile bed-load conditions, respectively. For the same density of solid particles, Einstein (1955) and Paintal (1971) carried out 15 and 37 sets of experimental observations, respectively, by varying the slope of the channel for different bed materials of bed-load condition. Smart and Jaeggi (1983) conducted 60 experiments of mobile bed load on one channel width by varying the slope for three conditions of grain size. Rickenman (1990) showed 46 more experiments with similar parameters as those by Smart and Jaeggi (1983), but for different channel slopes. Cao (1985) and Graf and Suszuka (1987) performed 56 and 106 experiments, respectively, on a channel with a width of 0.6 m by varying the slope for different bed materials for mobile bed conditions. Recking (2006) performed an experimental study on 79 different geometric and hydraulic conditions and even analyzed the experimental works of various other researchers. For measurement of velocity, Smart and Jaeggi (1983), Cao (1985), and Rickenmann (1991) used the salt velocity technique. Recking (2006) used an

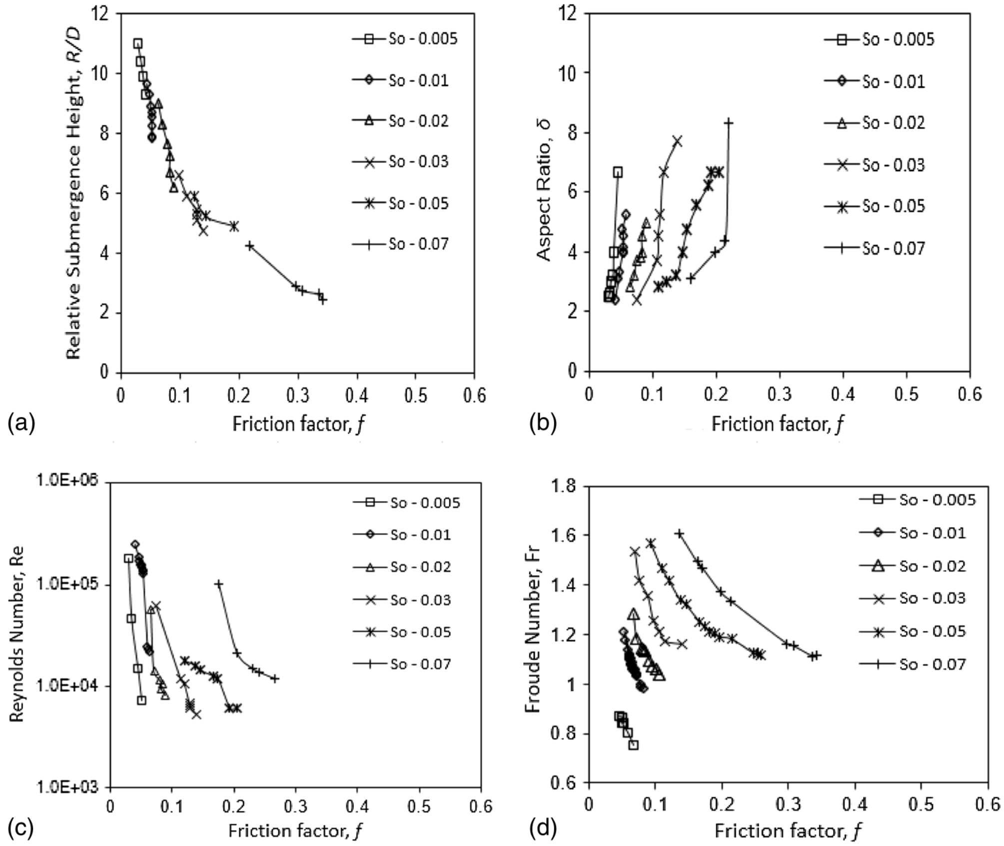

Fig. 1. Variations of friction factor f with respect to various influential parameters: (a) relative submergence height (R/D); (b) aspect ratio (δ); (c) Reynolds number (Re); and (d) Froude number (Fr).

image analysis technique and achieved a maximum error of 1.5% on the velocity calculation for the considered flow conditions. Appendix II presents the range for the various influencing dimensionless terms derived from the experimental data sets.

Selection of Influencing Dimensionless Terms

A wide range of data sets facilitate the analysis to observe the effect of various parameters on the friction factor. To ascertain the distinct effects of each of five selected dimensionless parameters, i.e., the longitudinal bed slope, S_o; relative submergence height (R/D); aspect ratio $(\delta = W/H)$; Reynolds number, Re; and Froude number, Fr on the flow resistance, the variation of friction factor with respect to the parameters are analyzed. For this purpose, the extensive set of data series was sorted based on increasing bed slope, and variation patterns of friction factor against selected dimensionless parameters for different bed slopes were analyzed. The variation pattern of output (i.e., friction factor) concerning respective dimensionless inputs at different bed slopes is shown in Figs. 1(a–d). While perceiving one parameter's effect on the friction factor for a particular slope, caution was exercised to observe that other parameters are not varying. This has been done to understand the trend of variation of the friction factor with considered dimensionless parameters. Fig. 1(a) shows the variation of friction factor with relative submergence height (R/D) and it can be seen that for lower bed slopes, there is not much change in the friction factor with (R/D); but for higher slopes, even a slight variation of (R/D) causes significant variations in the friction factor. For smaller slope values, there are little variations of the friction factor with aspect ratio, as shown in Fig. 1(b). However, for higher slope values, i.e., at $S_o = 0.05$ and 0.07, much variation of friction factor with the aspect ratio is noticed. Similar observations are made for higher bed-slope values on friction factor, which show strong dependency on Reynolds number as seen in Fig. 1(c). It is observed that there is a power-law variation of friction factor with respect to the Froude number, as seen in Fig. 1(d), where the friction factor increases with the decrease in Fr, and the variation is observed to be similar for different values of longitudinal bed slope.

Figs. 1(a–d) show the nonlinear variation of friction factor with respect to the influencing parameters for different values of bed slope, which is another influencing parameter. An extensive set of data has facilitated the analysis to observe the effect of various parameters on the friction factor. Thus, based on the preceding observation, a data-driven soft computing technique having the ability to analyze the nonlinear and complex relationship between multiple inputs and a single output was used to provide a reliable model for predicting flow resistance.

Application of Advanced Soft Computing Techniques

Advanced soft computing techniques based on artificial intelligence (AI) and machine learning are acquiring importance and being used in several water resources engineering fields for hydrological and hydraulic investigations. Shamshirband et al. (2020) used machine learning models for predicting a standardized streamflow index for hydrological drought studies. Fotovatikhah et al. (2018) presented a comprehensive survey about applying computational intelligence-based methods in flood management systems. Gholami et al. (2016) examined an ANN model in predicting the velocity and surface profile in a 90° open-channel bend. Wu and Chau (2013) compared several computing models based on ANN for rainfall prediction. Cheng et al. (2005) developed an adaptive-network-based fuzzy inference system (ANFIS) to forecast the long-term discharges reliably for hydropower reservoir management and scheduling. Cheng and Chua (2004) implemented a data-driven modeling technique for a real-time flood control management system for reservoirs in China.

ANN Modeling

An artificial neural network is considered a capable modeling technique, particularly for data sets having nonlinear relationships between multiple inputs and outputs. ANNs work on analyzing data sets for identification and training correlative patterns between input and output data pairs. Training is the most important part of ANN modeling, enabling it to generalize and predict outputs from new input data sets. In a network structure based on ANN, neurons are organized in a fully interconnected pattern in three layers: the input layer, the hidden layers, and the output layer. Neurons present in the input layer receive data from a data file. Neurons present in the output layer provide network response to the input data. Neurons present in the hidden layer perform the data processing by way of communication with other neurons. Three different layers form a pattern or network structure to derive a solution to the problem. The theory says that most functions can be approximated using a single hidden layer (Ripley 1996). The back-propagation (BP) training algorithm allows networks to adjust their hidden layers of neurons when the outcome does not match the desired output. The interrelation between the layers is an essential factor called weights for converting the input to impact the output. In an ANN, the weights are enhanced to limit predefined cost functions that drive the direction of the search for the optimal solution (Khuntia et al. 2018, 2019). With the implementation of the technique, the network is initially trained, and the target output of every output neuron is limited by specifying the weights and biases through the learning algorithm. The network learns by analyzing single data points, attaining a prediction for each record and adjusting the weights once it creates a wrong prediction. This is a repetitive process; therefore, the network moves forward to improve its predictions until any of the stopping criteria have been met. Like other data-driven modeling approaches, ANNs also have limitations because the quality of model output is primarily dependent on the quality of the input data. The implementation of ANNs is time consuming, making it computationally expensive; the basic drawback is that it is not always apparent how they are able to reach a solution, and due to this they have been often referred to as black boxes. But ANNs have a main advantage in that they are able to perform multivariate nonlinear regression analysis of multiple input data sets having complex and nonlinear relationships that are not adaptable for interpretation by conventional computational means.

ANN modeling is acquiring importance and being used as an advanced computational technique in several fields of water resources engineering. Shayya and Sablani (1998), Maier and Dandy (2000), ASCE Task Committee on Application of Artificial Neural Networks in Hydrology (2000a, b), Dolling and Varas (2002), Riad et al. (2004), Samani et al. (2007), and Sahu et al. (2011) discussed the implementation of ANN modeling for prediction of flow parameters. The ANN approach at present has become a widely accepted computational tool in many disciplines, including electrical engineering (Sharma et al. 2017), mechanical engineering (Parlak et al. 2006), and geology (Yuanyou et al. 1997).

Implementing Artificial Neural Networks

The present study proposes a flow friction law model by utilizing five input parameters in the network: longitudinal bed slope (S_o), relative submergence height (R/D), aspect ratio (δ), Reynolds number (Re), and Froude number (Fr), and one output parameter corresponding to the friction factor, f. Three-layer feed-forward back-propagation hierarchical networks with different architecture were designed using the Neural Network Toolbox of the MATLAB version R2020b platform. Levenberg-Marquardt's back-propagation training algorithm trainlm, which performs reasonably well for nonlinear regression, was used to implement ANN. The mean-square error (MSE) between the modeled and measured values was used as the performance function. The divide function dividerand was accessed at the time of training the network, which randomly partitioned the data into training and validation/testing subsets, with the ratio for training and testing/validation as 70% and 30%, respectively. Thus, out of the total 1,269 series of data sets, 888 were used as training data, and 381 were used for validation and testing purposes. Different architectures comprising single hidden-layer topology with a variation of numbers of neurons in input and hidden layer were used to assess the sensitivity of the model estimations to different combinations of input parameters as well as network structure on the output parameter corresponding to friction factor. The MAPE, MAE, MPE, RMSE, PBIAS, KGE, and I_d values between the predicted and the desired outputs were taken as the performance indicators to determine the input combination and network structure with optimal predictive capability. Table 1 indicates the performance indicators of the prospective ANN models for overall data sets. Higher MAPE, MAE, MPE, RMSE, and PBIAS, and lower KGE and I_d values are obtained when excluding any one of the selected input dimensionless terms. This signifies that each of the five input dimensionless terms has a significant influence on the friction factor. Results show that among the ANN models, Network 5-6-1 with all input combinations gave the best estimation with the MAPE, RMSE, MAE, MPE, PBIAS, KGE, and I_d values as 3.95, 0.004, 0.002, −0.18, −0.026, 0.997, and 0.997, respectively. Corresponding values for Networks 5-5-1 and 5-7-1 with all input combination were obtained as 8.28, 0.006, 0.004, −0.40, −0.370, 0.989, and 0.993, and 4.84, 0.004, 0.003, −2.63, 1.384, 0.984, and 0.996, respectively.

The network structure with five neurons in the input layer, six neurons in the hidden layer, and one neuron in the output layer designated 5-6-1 is observed to provide the best predictive performance because the coefficient of correlation during the training and testing stages was found to be high with low mean-square error value compared with that of other tested network structures.

Fig. 2(a) shows the comparison of predicted f versus the target f at the end of the training, and that the coefficient of determination (R^2) is 0.9975. This indicates that the training of the ANN model was proper. Fig. 2(b) shows the performance of the ANN model for f in testing data; the coefficient of determination (R^2) is 0.9976.

Validation of ANN Model for Unseen Data

The preceding implementation of the ANN technique used data sets randomly partitioned into training and validation/testing subsets. Thus, a profound approach was adopted to check whether the model can generalize from a training set to previously unseen data and continues to make reliable predictions. This was examined by evaluating the proposed ANN model's performance by dedicating data sets of a single experiment as unseen data sets for validation/checking while balance data sets were used for training and testing to determine the model. Four such combinations of data sets were considered by taking data sets from 12 out of 13 sets of experimental studies for the formulation of the ANN model, with all data sets belonging to a single set of experiment used each time for validation/checking. The results in terms of the coefficient of correlation (R) and MSE for actual and predicted values of the friction factor of a particular set of experiments obtained for each combination is given in Table 2. Fig. 3 shows mean square error (MSE) versus the number of data sets for f for all input combination and Network 5-6-1.

Table 1. Performance parameters of prospective ANN models for distinct input combination and network structures trained with Levenberg-Marquardt trainlm algorithm

Models with distinct input combinations of predictors	Network	MAPE (%)	RMSE	MAE	MPE (%)	PBIAS (%)	KGE	I_d
$f = \mathrm{fn}(R/D, W/H, \mathrm{Fr}, \mathrm{Re}, S_o)$	5-5-1	8.28	0.006	0.004	−0.40	−0.370	0.989	0.993
	5-6-1	3.95	0.004	0.002	−0.18	−0.026	0.997	0.997
	5-7-1	4.84	0.004	0.003	−2.63	1.384	0.984	0.996
$f = \mathrm{fn}(R/D, W/H, \mathrm{Fr}, \mathrm{Re})$	4-4-1	19.71	0.025	0.014	−4.03	0.460	0.921	0.922
	4-5-1	22.68	0.029	0.016	−0.64	−2.899	0.893	0.911
	4-6-1	18.95	0.024	0.013	−4.43	0.406	0.932	0.930
$f = \mathrm{fn}(R/D, Fr, R_e, S_o)$	4-4-1	9.983	0.011	0.006	−1.51	−0.265	0.936	0.932
	4-5-1	12.53	0.012	0.008	−0.61	−0.447	0.935	0.939
	4-6-1	9.97	0.011	0.006	−0.88	−0.289	0.925	0.932
$f = \mathrm{fn}(R/D, W/H, R_e, S_o)$	4-4-1	19.59	0.029	0.017	−8.70	3.282	0.878	0.904
	4-5-1	19.18	0.031	0.015	−3.19	−1.294	0.868	0.899
	4-6-1	24.60	0.034	0.017	−11.80	1.742	0.825	0.885
$f = \mathrm{fn}(R/D, W/H, Fr, S_o)$	4-4-1	10.86	0.008	0.005	−1.05	−0.711	0.937	0.938
	4-5-1	5.83	0.005	0.003	−1.22	0.407	0.934	0.936
	4-6-1	10.75	0.008	0.006	2.60	−3.043	0.931	0.938

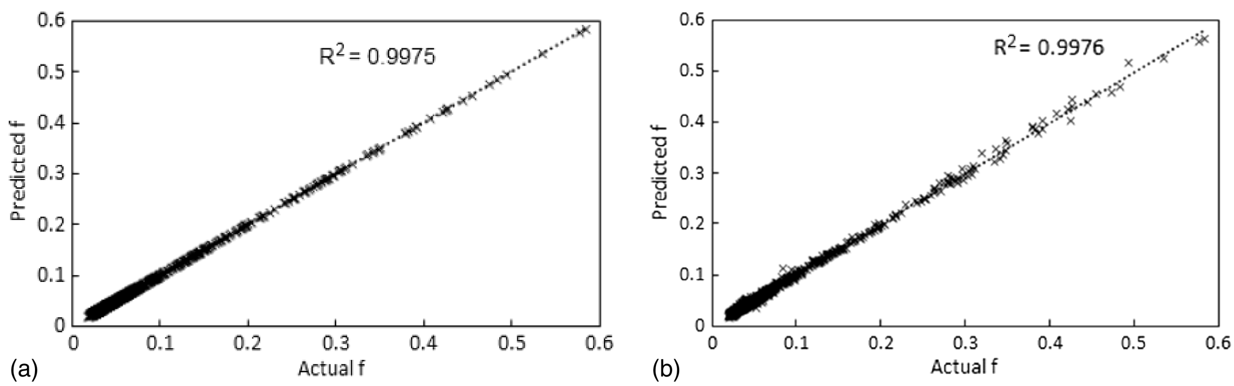

Fig. 2. Performance of ANN model for f with all input combinations and Network 5-6-1: (a) training stage; and (b) testing stage.

Table 2. Performance parameters of the ANN model for actual and predicted values of friction factor for validation of data sets taken from a single experiment

Serial No.	Data sets used for formulation	Data sets for validation	Coefficient of correlation (R)	MSE
1	1,190 data sets from 12 experiments	All 79 data sets of Recking (2006)	0.997	8.052×10^{-4}
2	1,226 data sets from 12 experiments	All 44 data sets of Bogardi and Yen (1939)	0.998	0.188×10^{-4}
3	1,233 data sets from 12 experiments	All 36 data sets of Paintal (1971)	0.989	0.345×10^{-4}
4	1,209 data sets from 12 experiments	All 60 data sets of Smart and Jaeggi (1983)	0.979	3.681×10^{-4}

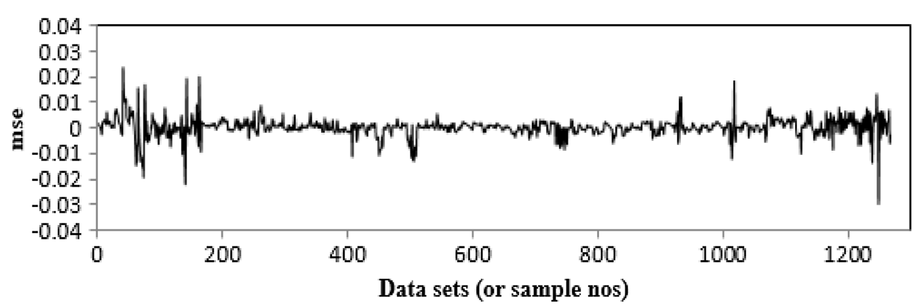

Fig. 3. Mean square error versus the number of data sets for f for all input combination and Network 5-6-1.

Table 3. Statistical error indexes and efficiency values of different models used for f

Model	MAPE (%)	RMSE	MAE	MPE (%)	PBIAS (%)	KGE	I_d
Recking et al. (2008)	18.94	0.093	0.063	−5.83	2.05	0.831	0.841
Recking (2006)	32.30	0.302	0.127	−28.45	37.78	−0.418	0.443
Cao (1985)	64.70	0.246	0.181	64.70	−70.92	−0.115	0.654
Meyer-Peter and Müller (1948)	9.67	0.007	0.005	−3.83	3.17	0.830	0.832
ANN model	3.95	0.004	0.002	−0.18	−0.026	0.997	0.997

From performance parameters obtained between observed and predicted values of friction factor for multiple validation data sets from single experimental sets, it was observed that the proposed ANN model is capable of generalization from a training set to previously unseen data and continues to make a reasonably precise prediction, signifying a qualitative modeling.

Results and Discussion

For comparative evaluation of the present ANN model with previous models, statistical performance measures in terms of error indexes such as MAPE, MAE, MPE, RMSE, PBIAS, efficiency values KGE, and I_d were computed using Eqs. (14)–(20). In general, the efficiency of any model can be proved when error indexes and efficiency values are near zero and close to 1, respectively. The performance of respective models of previous researchers were evaluated for their respective experimental data sets, while the proposed ANN model was subjected to evaluation for overall combined 1,269 data sets.

Table 3 lists the error indexes and efficiency values calculated for the present ANN model and the previous models. The performance of the proposed ANN model was found to be reasonably accurate with MAPE = 3.95%, MAE = 0.002, MPE = −0.18%, RMSE = 0.004, PBIAS = −0.026%, KGE = 0.997, and $I_d = 0.997$. As per statistical analysis, the present ANN model was found to be better with the lowest values of errors indexes and efficiency values very close to 1.

The coefficient of determination can also be considered a suitable statistical performance index to distinguish between the workability of different models. Fig. 4 plots the coefficient of determination (R^2) for different friction models. As can be observed in Fig. 4, the computed values of R^2 for the models of Recking et al. (2008), Recking (2006), Cao (1985), and Meyer-Peter and Müller (1948) were obtained as 0.753, 0.585, 0.365, and 0.736, respectively, whereas the proposed ANN model provides better results for a wide range of data sets with a high R^2 value of 0.998. The models by other researchers were evaluated, taking into consideration their respective experimental observations. The proposed ANN model considers the experimental observation of overall sets of data. Therefore, the proposed model has feasibility over a wider range of data sets.

The preceding analysis provided an overall understanding of the feasibility of different models. It is expected that the expressions for friction factor from the literature would result in a large error when used for individual data sets of other researchers. Therefore, to understand the acceptability of each model to various individual data sets, error analysis was carried out using the various expressions of friction factor mentioned in this paper. Such an analysis would provide a generalized understanding of the viability of each model. However, to assess the effectiveness of different models to various data sets, performance analysis was done by computing PBIAS, RMSE, and MAE for all the individual data sets. Computed values of MAE and RMSE were normalized with respect to the difference of maximum and minimum values of friction factor f of each individual data set to make it scale-free for reasonable comparison, called a normalized mean absolute error (NMAE) and normalized root-mean-square error (NRMSE), respectively.

Table 4 lists the PBIAS computed for individual data sets using expression of different models and the proposed ANN model. Results show that for the model suggested by Recking (2006) and Recking et al. (2008) the friction factors were underestimated and overestimated by 37.78% and 2.05%, respectively, while the proposed ANN model showed a negligible overestimation by 0.517% for the same data sets. Similarly, the friction factor according to the model suggested by Cao (1985) and Meyer-Peter and Müller (1948),

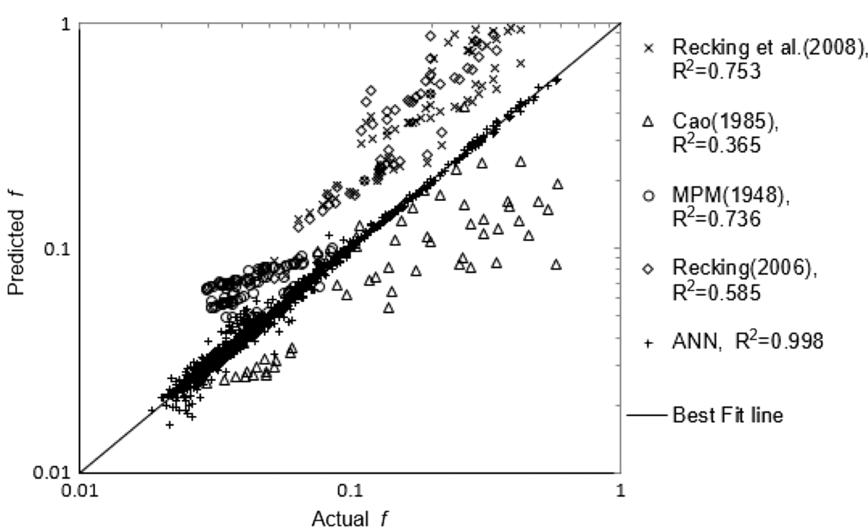

Fig. 4. Comparison of coefficient of determination from different models for friction factor f.

Table 4. PBIAS values for individual data sets for f

Data sets	Models: Meyer-Peter and Müller (1948)	Cao (1985)	Julien (2002)	Recking (2006)	Recking et al. (2008)	ANN model
Gilbert (1914)	14.07	−69.84	63.25	22.96	31.01	1.632
Cassey (1935)	33.46	−65.45	87.18	34.99	48.82	−1.978
Mavis (1937)	37.96	−60.05	115.41	60.90	79.25	0.468
Yang (1939)	34.01	−66.52	81.65	30.91	42.24	2.138
Bogardi and Yen (1939)	17.88	−68.15	69.82	56.01	57.54	0.753
Meyer-Peter and Müller (1948)	3.17	−71.59	53.48	13.19	25.26	−1.684
Einstein (1955)	10.25	−70.52	33.77	3.56	−1.93	−1.362
Paintal (1971)	11.35	−64.26	91.91	57.59	66.57	−0.087
Smart and Jaeggi (1983)	−40.20	−70.23	14.76	1.48	04.19	−0.376
Cao (1985)	−56.13	−70.92	50.79	96.49	98.60	0.117
Graf and Suszka (1987)	2.23	−63.39	95.78	69.73	76.94	−0.014
Rickenmann (1990)	−49.83	−70.85	22.55	36.70	22.39	−1.011
Recking (2006)	−65.68	−62.05	−5.57	37.78	2.05	−0.517

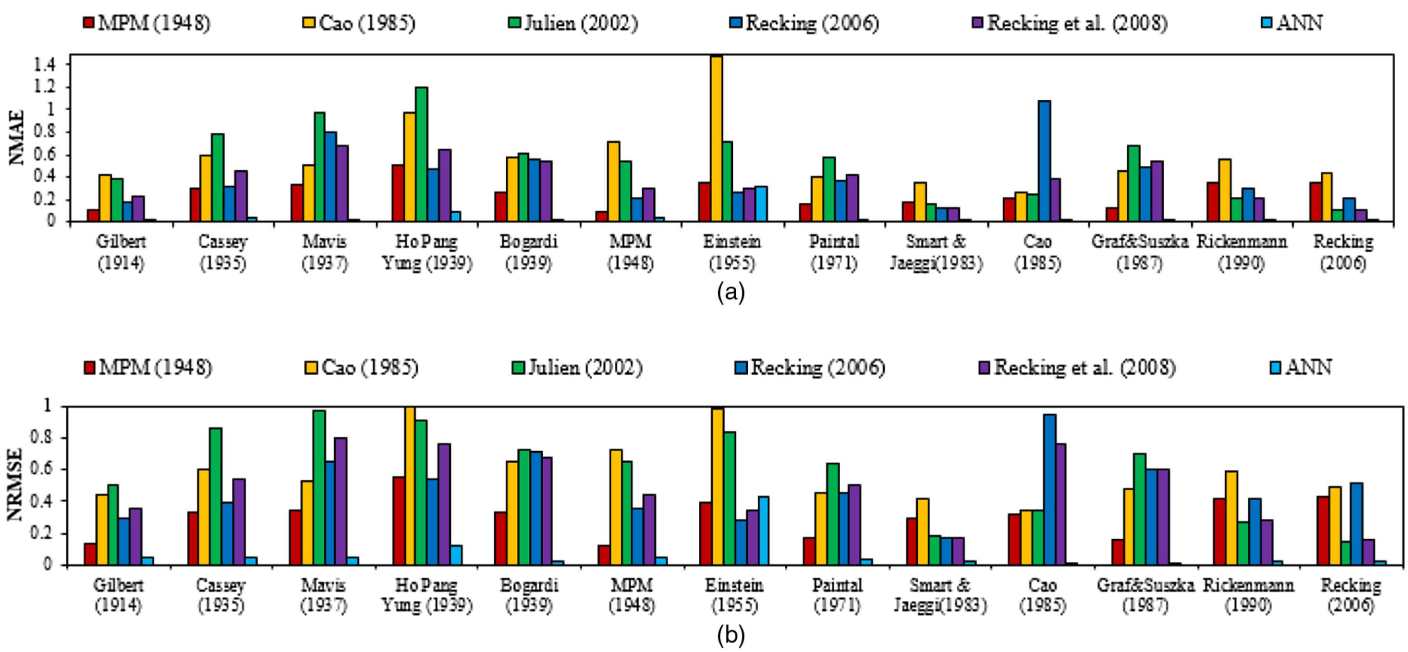

Fig. 5. Error analysis of different data sets: (a) comparison of NMAE of different data sets for studied models and ANN model; and (b) comparison of NRMSE of different data sets for studied models and ANN model.

when evaluated for their respective experimental data sets, were found to be overestimated and underestimated by 70.92% and 3.179%, respectively, while the proposed ANN model for the concerned data sets showed an insignificant underestimation and overestimation by 0.117% and 1.68%, respectively. The model proposed by Cao (1985) showed the overestimation of the friction factor in the range of 60% to 70% for all the individual data sets consistently. The model by Meyer-Peter and Müller (1948) estimated on both sides, but was reasonably better with its own data and the data sets of Graf and Suszka (1987) for which the PBIAS values are 3.179% and 2.233%, respectively. The PBIAS values calculated for the proposed ANN model varied from −1.978% to 2.138%, which indicate reasonably good performance for all individual data sets. Figs. 5(a and b) illustrate NRMSE and NMAE values for the individual data sets for the different friction factor models, where the proposed ANN model is observed to give negligible error close to zero. The proposed ANN model provided better flow resistance (i.e., friction factor f) than the other models because the ANN mapped the nonlinear relationship between the selected influenting flow parameters in complex flow conditions. This study used a wide range of data comprising the ranges of slope from 0.09% to 20%, relative submergence height from 1.10 to 260, aspect ratio from 1.17 to 100, Reynolds number from 2.16×10^3 to 1.44×10^6, and Froude number from 0.36 to 5.01. Further studies should be made to incorporate higher ranges of the parameters with turbulence properties to understand how the ANN model derived from flume experiments can prove beneficial for real field problems.

Conclusions

The present study proposed an ANN model to predict flow resistance in an open channel over a movable bed. The following conclusions are drawn from the research:

- A wide range of data sets was considered for observing the relationship between the independent parameters with the dependent parameter. The previous flow resistance models are restricted to the use of a single dimensionless parameter, i.e., relative submergence height (R/D), whereas the proposed ANN model considers five influencing dimensionless parameters such as the longitudinal bed slope, S_o; relative submergence height (R/D); aspect ratio (δ); Reynolds number (Re); and Froude number (Fr) as input parameters for the modeling.

- The trend and pattern of data match with predicted flow resistance, and the capability of prediction via ANN was demonstrated. The primary reason for a high degree of prediction accuracy lies in efficient nonlinear mapping between the inputs and output in ANN. The nonlinear relationship of geometrical, surface, and hydraulic input parameters with flow resistance in mobile bed channel is difficult to analyze with traditional prediction methodology. Additionally, the conventional techniques cannot take into account the real-life factors operating in the system because the interactions between sediment transport, bedform, and flow properties are very complicated processes.
- The performance of the ANN model was also shown in terms of MAE, RMSE, MAPE, PBIAS, KGE, and I_d values for overall data sets. The results were further verified for individual data sets of different researchers in terms of statistical parameters, i.e., PBIAS, NMAE, and NRMSE. The correlation plots for different methods show that the ANN model is fitted with greater accuracy with the coefficient of determination as 0.998, whereas the same for the models of Recking et al. (2008) and Meyer-Peter and Müller (1948) were 0.753 and 0.736, respectively.
- The empirical equations suggested by Recking et al. (2008) and Meyer-Peter and Müller (1948) were observed to perform well in specific ranges of experimental conditions only. Compiling all the influencing geometric and hydraulic parameters, the ANN provided the best flow resistance value. From the statistical analysis, it can be concluded that the other models do not efficiently predict the flow resistance as compared to the ANN model. The proposed ANN model was validated with multiple sets of unseen data from the same distribution and reasonably accurate results were obtained. Thus, it can be inferred that the ANN model is capable of generalization for interpreting information different to that of the training data sets and predicting the output or trends based on what they have previously seen.
- This study uses a wide range of data comprising the ranges of slope from 0.09% to 20%, relative submergence height from 1.10 to 260, aspect ratio from 1.17 to 100, Reynolds number from 2.16×10^3 to 1.44×10^6, and Froude number from 0.36 to 5.01. Further studies should be performed to incorporate a higher range of the parameters of a mobile bed channel to examine the ability of extrapolation of the network and also to incorporate the turbulence anisotropy and curvature effects that could help to understand how the ANN model derived from flume experiments in bed-load transport conditions can prove more beneficial for real field problems in case of nonprismatic meandering channels.

Appendix I. Geometric and Hydraulic Flow Conditions for the Experimental Data Sets

Data sets	Discharge, Q (m^3/s)	Slope, S_o (%)	Width (m)	Depth of flow (m)	Grain mean diameter, D (mm)	Density of solid particle	Sediment concentration (g/m^3)
Gilbert (1914)	0.003–0.032	0.34–2.25	0.20–0.59	0.02–0.17	0.51, 3.17, 7.01, 4.94	2.65	315–33,000
Cassey (1935)	0.001–0.099	0.12–0.52	0.40	0.01–0.29	1.0, 2.5	2.65, 2.81	1.6–2,722
Mavis (1937)	0.002–0.078	0.14–1.01	0.89	0.01–0.13	1.4, 2.0, 3.1, 3.7, 4.2	2.66	0.7–2,361.9
Bogardi and Yen (1939)	0.016–0.064	1.04–2.45	0.30–0.82	0.03–0.14	6.8, 9.0, 10.3, 15.2	2.63, 2.61	6.54–1,010
Yang (1939)	0.003–0.069	0.09–0.50	0.39	0.04–0.26	1.4, 2.0, 3.1, 4.4, 6.0, 6.3	2.45, 2.5, 2.64, 2.66, 2.7	0.1–351
Meyer-Peter and Müller (1948)	0.001–4.613	0.13–2.27	0.15–2.00	0.01–1.09	1.17–28.65	2.66	0.7–2,361
Einstein (1955)	0.074–0.083	1.24–2.58	0.31	0.11–0.14	0.27, 0.94, 1.30	2.65	2,543–52,238
Paintal (1971)	0.02605–0.25484	0.13-1.00	0.91–0.92	0.08–0.21	2.50, 7.95, 22.20	2.65	0.19–348.23
Smart and Jaeggi (1983)	0.005–0.03	3.00–20	0.20	0.02–0.08	2.0, 4.2, 10.5	2.67, 2.68	4,666–830,666
Cao (1985)	0.015–0.250	0.50–9.00	0.60	0.03–0.25	11.5, 2.2, 44.3	2.65, 2.75	9.39–71,103
Graf and Suszuka (1987)	0.040–0.205	0.5–2.5	0.60	0.07–0.26	12.2, 23.5	2.72	1.83–26,66
Rickenmann (1990)	0.01–0.03	7.00–20.	0.20	0.03–0.09	10	2.68	39,333–1,356,000
Recking (2006)	0.006-0.015	1.00–9.00	0.10–0.25	0.01–0.07	2.3, 4.9, 9.0, 12.5	2.6	4,000–99,111

Appendix II. Influencing Parameters for the Experimental Data Sets Undertaken in the Study

Data sets	Slope, S_o (%)	Relative submergence height $\left(\frac{R}{D}\right)$	Aspect ratio, $\delta(W/H)$	Froude number, Fr	Reynold's number, Re
Gilbert (1914)	0.34–2.25	4.93–115.75	1.17–33.33	0.72–2.17	12,037–82,939
Cassey (1935)	0.12–0.52	8.70–78.80	1.38–50.00	0.43–0.89	2,161–114,437
Mavis (1937)	0.14–1.01	3.04–11.38	6.3–100.0	0.53–1.21	2,930–80,607
Bogardi and Yen (1939)	1.04–2.45	3.20–9.13	2.17–25.00	0.85–1.58	26,599–100,434
Yang (1939)	0.09–0.50	10.75–71.81	1.51–11.11	0.36–0.83	8,109–100,889
Meyer-Peter and Müller (1948)	0.13–2.27	5.37–73.01	2.32–8.33	0.38–1.41	3,965–1,439,179
Einstein (1955)	1.24–2.58	48.8–260.0	2.17–2.85	1.61–2.15	151,445–158,619

J. Hydrol. Eng., 2021, 26(5): 04021015

Appendix II. (*Continued.*)

Data sets	Slope, S_o (%)	Relative submergence height $\left(\frac{R}{D}\right)$	Aspect ratio, $\delta(W/H)$	Froude number, Fr	Reynold's number, Re
Paintal (1971)	0.13–1.00	4.79–51.30	4.34–14.28	0.51–0.99	27,939–217,096
Smart and Jaeggi (1983)	3–20	2.70–18.27	2.38–10.00	1.32–5.01	21,427–109,396
Cao (1985)	0.5–9.0	1.10–10.97	2.38–16.67	0.78–1.58	25,156–253,508
Graf and Suszuka (1987)	0.5–2.5	3.04–11.38	2.32–8.33	0.77–1.26	60,413–208,195
Rickenmann (1990)	7–20	2.40–4.61	2.32–6.25	1.45–3.76	38,705–108,996
Recking (2006)	1–9	1.50–20.38	2.85–8.33	0.93–1.75	5,159–42,000

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article. For the data sets used in this article, please refer to Appendixes I and II.

Acknowledgments

The authors wish to express sincere thanks to the anonymous editors and reviewers for their time in effort in reviewing and offering constructive suggestions that improved the manuscript. We are thankful to Dr. Kamalini Devi, Associate Professor, Vidya Jyothi Institute of Technology (VJIT) Hyderabad, and Dr. Shreedevi Moharana, NPDF, Indian Institute of Technology (IIT) Hyderabad, for their expertise and suggestions in revising the manuscript. The authors would like to sincerely thank all the previous researchers for the valuable experimental data sets. All the researchers listed in the references are also sincerely acknowledged.

References

ASCE Task Committee on Application of Artificial Neural Networks in Hydrology. 2000a. "Artificial neural networks in hydrology. I: Preliminary concepts." *J. Hydrol. Eng.* 5 (2): 115–123. https://doi.org/10.1061/(ASCE)1084-0699(2000)5:2(115).

ASCE Task Committee on Application of Artificial Neural Networks in Hydrology. 2000b. "Artificial neural networks in hydrology. II: Hydrologic applications." *J. Hydrol. Eng.* 5 (2): 124–137. https://doi.org/10.1061/(ASCE)1084-0699(2000)5:2(124).

Bathurst, J. C., W. H. Graf, and H. H. Cao. 1982. "Bedforms and flow resistance in steep gravel-bed channels." In *Mechanism of sediment transport*, 215–221. Boca Raton, FL: CRC Press.

Bogardi, J., and C. H. Yen. 1939. *Tractation of pebbles by flowing water*. Ames, IA: State Univ. of Iowa.

Campbell, L., I. McEwan, V. Nikora, D. Pokrajac, M. Gallagher, and C. Manes. 2005. "Bed-load effects on hydrodynamics of rough-bed open-channel flows." *J. Hydraul. Eng.* 131 (7): 576–585. https://doi.org/10.1061/(ASCE)0733-9429(2005)131:7(576).

Cao, H. H. 1985. "Resistance hydraulique d'un lit à gravier mobile à pente raide; étude expérimentale." Ph.D. thesis, Département de Génie Civil, Ecole PolytechniqueFederale de Lausane.

Carbonneau, P. E., and N. E. Bergeron. 2000. "The effect of bedload transport on mean and turbulent flow properties." *Geomorphology* 35 (3–4): 267–278. https://doi.org/10.1016/S0169-555X(00)00046-5.

Casey, H. J. 1935. *Uber Geschiebebewegung. Mitt. der Preuss. Versuchanst*. Berlin: Wasserbauund Schifiau.

Cheng, C. T., and K. W. Chau. 2004. "Flood control management system for reservoirs." *Environ. Modell. Software* 19 (12): 1141–1150. https://doi.org/10.1016/j.envsoft.2003.12.004.

Cheng, C. T., J. Y. Lin, Y. G. Sun, and K. Chau. 2005. "Long-term prediction of discharges in Manwan hydropower using adaptive-network-based fuzzy inference systems models." In *Proc., Int. Conf. on Natural Computation*, 1152–1161. Berlin: Springer.

Dolling, O. R., and E. A. Varas. 2002. "Artificial neural networks for streamflow prediction" *J. Hydraul. Res.* 40 (5): 547–554. https://doi.org/10.1080/00221680209499899.

Einstein, H. A. 1955. *Effect of heavy sediment concentration near the bed on velocity and sediment distribution*. Berkeley, CA: Univ. of California.

Ferro, V. 2018. "Flow resistance law under equilibrium bedload transport conditions" *Flow Meas. Instrum.* 64 (Dec): 1–8. https://doi.org/10.1016/j.flowmeasinst.2018.10.008.

Fotovatikhah, F., M. Herrera, S. Shamshirband, K. W. Chau, S. Faizollahzadeh Ardabili, and M. J. Piran. 2018. "Survey of computational intelligence as basis to big flood management: Challenges, research directions and future work." *Eng. Appl. Comput. Fluid Mech.* 12 (1): 411–437. https://doi.org/10.1080/19942060.2018.1448896.

Gao, P., and A. D. Abrahams. 2004. "Bedload transport resistance in rough open-channel flows." *Earth Surf. Process. Landforms* 29 (4): 423–435. https://doi.org/10.1002/esp.1038.

Garcia, M., ed. 2008. *Sedimentation engineering: Processes, measurements, modeling, and practice*. Manual of Practice 110. Reston, VA: ASCE.

Gholami, A., H. Bonakdari, A. H. Zaji, S. Ajeel Fenjan, and A. A. Akhtari. 2016. "Design of modified structure multi-layer perceptron networks based on decision trees for the prediction of flow parameters in 90° open-channel bends." *Eng. Appl. Comput. Fluid Mech.* 10 (1): 193–208. https://doi.org/10.1080/19942060.2015.1128358.

Gilbert, G. K. 1914. Vol. 86 of *The transportation of debris by running water*. Washington, DC: USGS, US Government Printing Office.

Graf, W. H., and L. Suszka. 1987. "Sediment transport in steep channels." *J. Hydroscience Hydraul. Eng.* 5 (1): 11–26.

Hou, J., C. Zhang, D. Wang, F. Li, Z. Yu, and Q. Zhou. 2019. "Fixed-bed and mobile-bed resistance of channels with steep gradients in mountainous areas." *Water* 11 (4): 681. https://doi.org/10.3390/w11040681.

Hu, S., and A. D. Abrahams. 2004. "Resistance to overland flow due to bedload transport on plane mobile beds." *Earth Surf. Processes Landforms J. Br. Geomorphological Res. Group* 29 (13): 1691–1701. https://doi.org/10.1002/esp.1123.

Hu, S., and A. D. Abrahams. 2005. "The effect of bed mobility on resistance to overland flow." *Earth Surf. Processes Landforms J. Br. Geomorphological Res. Group* 30 (11): 1461–1470. https://doi.org/10.1002/esp.1223.

Julien, P. Y. 2002. *River mechanics*. New York: Cambridge University Press.

Keulegan, G. B. 1938. "Laws of turbulent flow in open channels." *J. Res. Nat. Bur. Stand.* 21: 707–741. https://doi.org/10.6028/jres.021.039.

Khuntia, J. R., K. Devi, and K. K. Khatua. 2018. "Boundary shear stress distribution in straight compound channel flow using artificial neural network." *J. Hydrol. Eng.* 23 (5): 04018014. https://doi.org/10.1061/(ASCE)HE.1943-5584.0001651.

Khuntia, J. R., K. Devi, and K. K. Khatua. 2019. "Flow distribution in a compound channel using an artificial neural network." *Sustainable Water Resour. Manage.* 5 (4): 1847–1858. https://doi.org/10.1007/s40899-019-00341-2.

Kumar, B. 2011. "Flow resistance in alluvial channel." *Water Resour.* 38 (6): 745–754. https://doi.org/10.1134/S009780781106008X.

Maier, H. R., and G. C. Dandy. 2000. "Neural networks for the prediction and forecasting of water resources variables: A review of modelling issues and applications." *Environ. Modell. Software* 15 (1): 101–124. https://doi.org/10.1016/S1364-8152(99)00007-9.

Manning, R. 1891. "On the flow of water in open channels and pipes." *Inst. Civ. Eng. Irel. Trans.* 20: 161–207.

Mavis, F. T., T. Liu, and E. Soucek. 1937. *The transportation of detritus by flowing water.* Iowa City, IA: Univ. of Iowa.

Meyer-Peter, E., and R. Müller. 1948. "Formulas for bedload transport." In *Proc., IAHSR 2nd Meeting*, Stockholm, Sweden: International Association for Hydro-Environment Engineering and Research.

Omid, M. H., M. Karbasi, and J. Farhoudi. 2010. "Effects of bedload movement on flow resistance over bed forms." *Sadhana* 35 (6): 681. https://doi.org/10.1007/s12046-010-0045-6.

Paintal, A. S. 1971. "Concept of critical shear stress in loose boundary open channels." *J. Hydraul. Res.* 9 (1): 91–113. https://doi.org/10.1080/00221687109500339.

Parlak, A., Y. Islamoglu, H. Yasar, and A. Egrisogut. 2006. "Application of artificial neural network to predict specific fuel consumption and exhaust temperature for a diesel engine." *Appl. Therm. Eng.* 26 (8–9): 824–828. https://doi.org/10.1016/j.applthermaleng.2005.10.006.

Recking, A. 2006. *An experimental study of grain sorting effects on bedload*. Rep. No. 2006-ISAL-00113. Ph.D. thesis, Dept. of Civil Engineering, Institut National des Sciences Appliquées de Lyon.

Recking, A., P. Frey, A. Paquier, P. Belleudy, and J. Y. Champagne. 2008. "Bed-load transport flume experiments on steep slopes." *J. Hydraul. Eng.* 134 (9): 1302–1310. https://doi.org/10.1061/(ASCE)0733-9429(2008)134:9(1302).

Riad, S., J. Mania, L. Bouchaou, and Y. Najjar. 2004. "Rainfall-runoff model usingan artificial neural network approach." *Math. Comput. Modell.* 40 (7–8): 839–846. https://doi.org/10.1016/j.mcm.2004.10.012.

Rickenmann, D. 1990. "Bedload transport capacity of slurry flows at steep slopes." Ph.D. dissertation, Dept. of Civil Engineering, ETH Zurich.

Rickenmann, D. 1991. "Hyperconcentrated flow and sediment transport at steep slopes." *J. Hydraul. Eng.* 117 (11): 1419–1439. https://doi.org/10.1061/(ASCE)0733-9429(1991)117:11(1419).

Ripley, B. D. 1996. *Neural networks and pattern recognition*. New York: Cambridge University Press.

Sahu, M., K. K. Khatua, and S. S. Mahapatra. 2011. "A neural network approach for prediction of discharge in straight compound open channel flow." *Flow Meas. Instrum.* 22 (5): 438–446. https://doi.org/10.1016/j.flowmeasinst.2011.06.009.

Samani, N., M. Gohari-Moghadam, and A. A. Safavi. 2007. "A simple neural network model for the determination of aquifer parameters." *J. Hydrol.* 340 (1–2): 1–11. https://doi.org/10.1016/j.jhydrol.2007.03.017.

Shamshirband, S., S. Hashemi, H. Salimi, S. Samadianfard, E. Asadi, S. Shadkani, and K. W. Chau. 2020. "Predicting standardized streamflow index for hydrological drought using machine learning models." *Eng. Appl. Comput. Fluid Mech.* 14 (1): 339–350. https://doi.org/10.1080/19942060.2020.1715844.

Sharma, P., D. Shrivastava, S. More, P. Tripathi, and S. Jain. 2017. "Application of soft computing in electrical engineering." *Int. J. Eng. Res. Technol.* 5 (1): 1–6.

Shayya, W. H., and S. S. Sablani. 1998. "An artificial neural network for non-iterative calculation of the friction factor in pipeline flow." *Comput. Electron. Agric.* 21 (3): 219–228. https://doi.org/10.1016/S0168-1699(98)00032-5.

Smart, G., and M. Jaeggi. 1983. *Sediment transport on steep slopes*. Zurich, Switzerland: Mitteilungen der Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie.

Song, T., Y. M. Chiew, and C. O. Chin. 1998. "Effect of bed-load movement on flow friction factor." *J. Hydraul. Eng.* 124 (2): 165–175. https://doi.org/10.1061/(ASCE)0733-9429(1998)124:2(165).

Wiberg, P. L., and D. M. Rubin. 1989. "Bed roughness produced by saltating sediment." *J. Geophys. Res. Oceans* 94 (4): 5011–5016. https://doi.org/10.1029/JC094iC04p05011.

Wu, C. L., and K. W. Chau. 2013. "Prediction of rainfall time series using modular soft computing methods." *Eng. Appl. Artif. Intell.* 26 (3): 997–1007. https://doi.org/10.1016/j.engappai.2012.05.023.

Yang, H. P. 1939. *Dependence of the sediment movement on the grain shape and the temperature*. Berlin: Preuss. Versuchsanst. fur Wasserbau und Schifibau.

Yang, S.-Q., and S.-Y. Lim. 2003. "Total load transport formula for flow in alluvial channels." *J. Hydraul. Eng.* 129 (1): 68–72. https://doi.org/10.1061/(ASCE)0733-9429(2003)129:1(68).

Yuanyou, X., X. Yanming, and Z. Ruigeng. 1997. "An engineering geology evaluation method based on an artificial neural network and its application." *Eng. Geol.* 47 (1–2): 149–156. https://doi.org/10.1016/S0013-7952(97)00015-X.